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$1-p=1-2^4/6^3=1/100177688=P$, chance that D will throw less than 52.

P^2 =chance that D and E will both throw less.

$\therefore aP^2=C$'s expectation on the supposition that C wins and no ties.

49. Proposed by B. F. FINKEL, A. M., M. Sc., Professor of Mathematics and Physics, Drury College, Springfield, Missouri.

A square whose side is $2a$ and an equilateral triangle whose altitude is $3a$ are fastened together at their centers, but otherwise free to move. If they are thrown on a floor at random, what is the average area common to both?

Solution by HENRY HEATON, M. Sc., Atlantic, Iowa, and the PROPOSER.

In the figure let O be the common center of the square and the triangle.

Then $OK=ON=OH=OM=OL=OP=OI=a$.

Let the triangle $KON=2\theta$.

Then $\angle NOH=\frac{1}{2}\pi-2\theta$, $\angle HOM=\frac{3}{2}\pi-(\frac{1}{2}\pi-2\theta)=\frac{1}{2}\pi+2\theta$, $\angle MOL=\frac{1}{2}\pi-(\frac{1}{2}\pi+2\theta)=\frac{1}{2}\pi-2\theta$, $\angle LOP=\frac{3}{2}\pi-(\frac{1}{2}\pi-2\theta)=\frac{1}{2}\pi+2\theta$, and $\angle POI=\frac{1}{2}\pi-(\frac{1}{2}\pi+2\theta)=\frac{1}{2}\pi-2\theta$.

Area of surface, $KONQ$, $=a^2 \tan \theta$;

area of surface, $NOHW$, $=a^2 \tan(\frac{1}{2}\pi-\theta)$;

area of surface, $HOMV$, $=a^2 \tan(\frac{1}{2}\pi+\theta)$;

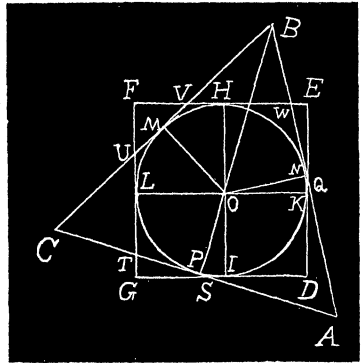
area of surface, $MOLU$, $=a^2 \tan(\frac{1}{2}\pi-\theta)$;

area of surface, $KOPT$, $=a^2 \tan(\frac{1}{2}\pi+\theta)$;

area of surface, $POIS$, $=a^2 \tan(\frac{1}{2}\pi-\theta)$;

area of square, $OKDI$, $=a^2$.

Hence the area common to the square and triangle is



$$S=a^2[1+\tan\theta+\tan(\frac{1}{2}\pi-\theta)+\tan(\frac{1}{2}\pi+\theta)+\tan(\frac{1}{2}\pi-\theta)+\tan(\frac{1}{2}\pi+\theta)+\tan(\frac{1}{2}\pi-\theta)].$$

The positions for $\theta > \frac{1}{2}\pi$ are exact repetitions of those for $\theta < \frac{1}{2}\pi$.

Hence the required average area is

$$A=\int_0^{\frac{1}{2}\pi} Sd\theta \div \int_0^{\frac{1}{2}\pi} d\theta=\frac{12a^2}{\pi} \int_0^{\frac{1}{2}\pi} \left[1+\tan\theta+\tan(\frac{1}{2}\pi-\theta)+\tan(\frac{1}{2}\pi+\theta)+\right.$$

$$\left.\tan(\frac{1}{2}\pi-\theta)+\tan(\frac{1}{2}\pi+\theta)+\tan(\frac{1}{2}\pi-\theta)\right]d\theta=a^2\left[1+\frac{12}{\pi}\log 2\right].$$

This problem was also solved in a very excellent manner by G. B. M. Zerr.

50. Proposed by G. B. M. ZERR, A. M., Ph. D., Texarkana, Arkansas.

Find (1), the average length of all straight lines having a given direction, between 0 and a ; (2), the average length of chords drawn from one extremity of the diameter a of a semi-circle to all points in the semi-circumference; and (3), find the average area of all triangles formed by a straight line of constant length a sliding between two straight lines at right angles.

Solution by HENRY HEATON, M. Sc., Atlantic, Iowa.

(1). Let x = length of one of the straight lines. Then the required average is $\int_0^a x dx \div \int_0^a dx = \frac{1}{2}a$.

(2). Let θ = angle between the chord and diameter. Then the length of the chord is $2a \cos \theta$, and the average length of all chords is

$$2a \int_0^{\frac{1}{2}\pi} \cos \theta d\theta \div \int_0^{\frac{1}{2}\pi} d\theta = 4a/\pi.$$

(3). Let θ = the angle between the sliding line and one of the fixed ones. The area of the triangle is $(\frac{1}{2}a^2) \sin \theta \cos \theta$. The average area of all such triangles depends upon circumstances. If the areas be taken at equal angular intervals, the required average is $A_1 = \int_0^{\frac{1}{2}\pi} (\frac{1}{2}a^2) \sin \theta \cos \theta d\theta \div \int_0^{\frac{1}{2}\pi} d\theta = a^2/2\pi$.

If the areas be taken at equal intervals as measured on one of the fixed lines along which the end of the sliding line moves, the average area is

$$A_2 = \int_0^{\frac{1}{2}\pi} (\frac{1}{2}a^2) \sin \theta \cos \theta d(a \sin \theta) \div \int_0^{\frac{1}{2}\pi} d(a \sin \theta) = \frac{1}{2}a^2 \int_0^{\frac{1}{2}\pi} \sin \theta \cos \theta d\theta = \frac{1}{4}a^2.$$

This is but a repetition of Problem 26, about which there has been so much controversy. In the absence of a distinct statement as to the intervals at which the areas shall be taken I see no reason for preferring either of the above solutions to the other.

The editor quotes me as saying that there is no correct solution of the problem. I must have failed in expressing myself clearly, for my position is that in such problems no solution can be considered a full one that does not discuss all possible cases.

51. Proposed by G. B. M. ZERR, A. M., Ph. D., Texarkana, Arkansas.

Three points are taken at random in a sphere and a plane passed through them. Find the average volume of the segment cut off from the sphere.

Solution by the PROPOSER.

Let A, B, C be the three random points, EF the diameter of the section of the sphere made by a plane through A, B, C ; M the center of this section; O the center of the sphere; OG a line such that AB is parallel to the plane MOG .

Let $OE = r, MA = x, AB = y, AC = z, \angle EOM = \theta, \angle BAM = \varphi, \angle CAM = \psi, \angle MOG = \lambda$, the angle the plane MOG makes with some fixed plane through $OG = \rho$.

The element of the sphere at A is $r \sin \theta d\theta \cdot 2\pi x dx$; at $B, y^2 dy d\varphi d\lambda$; at $C, \sin(\varphi + \psi) \sin \lambda z^2 dz d\psi d\rho$.

The limits of θ are 0 and $\frac{1}{2}\pi$; of $x, 0$ and $r \sin \theta$, and tripled; of $\varphi, -\frac{1}{2}\pi$ and $+\frac{1}{2}\pi$; of $\psi, -\varphi$ and $\frac{1}{2}\pi$, and doubled; of $\lambda, 0$ and π ; of $\rho, 0$ and 2π ; of $y, 0$ and $2x \cos \varphi$; of $z, 0$ and $2x \cos \psi$.

